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Study of Sensitivity of Numerically Simulated Turbulent Flow Field to Initial Conditions

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STUDY OF SENSITIVITY OF NUMERICALLY SIMULATED TURBULENT FLOW FIELD TO INITIAL CONDITIONS

1 Introduction

The aim of this study is to see how far the turbulence flow field is sensitive to initial conditions when it is simulated numerically by solving the Navier-Stokes equations. It is known experimentally that the turbulent flow fields are quite robust and that is exactly why the experiments involving the turbulent fields are repeatable. Hence one has to bother only about the boundary conditions but not worry how exactly a wind tunnel or a flow facility is started. Even though this is the case, one cannot take for granted that when a turbulent flow field is simulated by solving the Navier-Stokes equations the flow field is independent of the initial conditions. This is because so little is known about the uniqueness of the three-dimensional Navier-Stokes equations [4]. Further, these equations are discretised and solved numerically to add to the uncertainty. It is also known that [2] the smallest scales or the so called Kolmogorov microscales are not resolved in the simulation.

Hence it is imperative to see if such a simulated flow field is dependent on the initial conditions. If it so turns out, then one has a sticky problem in hand. Checking independence from initial conditions is an involved excercise. Also it is not known how to select the initial conditions except that this velocity field has to be solenoidal for an incompressible fluid (like the present one) and should satisfy the boundary conditions. Even though one can choose the initial conditions in an infinite number of ways, we have chosen in this study only two sets of initial conditions sufficiently different and compared the corresponding flow fields. We stop at that since these calculations are expensive. But this should suffice for the present purpose. There is another difficulty while comparing two fields that are unsteady and do not coincide exactly with each other. Hence a method has to be devised to see if these two fields are equivalent.

The problem studied here is that of flow in a three-dimensional cavity where the fluid is set into motion by moving one of the walls. As a preliminary exercise the case of Re = 1,000 was studied with two different initial conditions. Here the flow reaches an asymptotic steady state and hence the two solutions can be checked if they are identical. Then we take up the case Re = 10,000, again with two initial conditions and check if the two flow fields obtained are statistically equivalent.

2 The Computational Procedure

The flow in a parallelepiped of dimension l_x, l_y, l_z starts from rest due to the motion of the lid along the y-axis with a uniform velocity v_0. All lengths are non-dimensionalised by l_y, velocities by v_0 etc. Then,

$$
\begin{aligned}
(X, Y, Z) &= \frac{(x, y, z)}{l_y} \\
(L_x, L_y, L_z) &= \left(\frac{l_x}{l_y}, 1, \frac{l_z}{l_y}\right) \\
\vec{V} &= (U, V, W) = \frac{\vec{v}}{v_0} \\
\tau &= \frac{t v_0}{l_y} \qquad \text{Time} \\
P &= \frac{p}{\rho v_0^2} \qquad \text{Pressure} \\
Re &= \frac{v_0 l_y}{\nu} \qquad \text{Reynolds number}
\end{aligned}
$$

The top moving lid is located at $X = 0$ and since the cavity is assumed to be cubical, $L_x = L_y = L_z = 1$. The Navier-Stokes equations and the Poisson equation for pressure in their non-dimensional form are solved for U, V, W and P by a finite difference scheme. In this scheme convective terms are approximated by a third order upwind scheme and all the other spatial derivatives by the second order central difference scheme. Euler explicit scheme is used for time marching and the wall boundary conditions are approximated numerically by a second order scheme. The computer programme was parallelised and run on FLOSOLVER, the parallel computer of N.A.L. Parts of the computations were done on the CONVEX C3820 computer of CMMACS. All the results reported here were obtained on an $(84 * 84 * 84)$ grid.

For the details of computational procedure one can refer to [1].

Two numerical experiments are done. In the first one Re = 1,000 case is computed using two different initial conditions: $IC1$ is the zero initial condition case where the flow starts from rest; $IC2$ corresponds to flow computed for Re =3,200 where the flow is unsteady and stationary. Thus $IC2$ automatically satisfies the divergence free condition and is sufficiently different from $IC1$. In the second numerical experiment the case of Re = 10,000 is studied and again with the same two sets of initial conditions, $IC1$ and $IC2$.

To compare the two flow fields obtained for $Re = 10{,}000$ we compare the mean velocity and Reynolds stress profiles along selected lines and velocity spectra at some points. Since the signal length used for finding the statistical averages cannot be infinite, only an approximate agreement of the statistical quantities from the two flow fields can be expected. To help to decide whether these two fields are close enough to each other, a convergence study is undertaken in each of the fields separately. A particular length of the signal is taken and the statistical averages are found and then the signal length was increased to twice and again to three times the original length. By comparing and looking at the convergence trends of the statistical averages for these three lengths one can see what to expect even if the signals were to come from the same source.

3 Results and Discussion

As mentioned before two numerical experiments have been performed. In the first experiment convergence to an expected steady state starting from two different initial conditions is studied. In the second experiment convergence to a statistically stationary state at $Re = 10{,}000$ starting from these two initial conditions is studied. These are discussed below.

3.1 Convergence to a Steady State

In this numerical experiment $Re = 1{,}000$ is selected for study. This Reynolds number was selected since the flow is known to reach an asymptotic steady state at this value, the critical Reynolds number at which the flow becomes unsteady being around 2,000. Thus the selected value of $Re = 1{,}000$ is well below this critical value but at the same time it is moderately high and is a fairly difficult case to calculate numerically.

The flow was first calculated starting from rest and setting the lid into motion with a steady nondimensional velocity of unity along the Y-direction. This initial condition is called $IC1$. The flow at $Re = 3{,}200$ was studied earlier and was readily available as initial condition $IC2$ and was used in this and the next numerical experiment. At a particular value of time τ (here $\tau = 297.1$, the duration upto which $Re = 3{,}200$ case was computed) Re was changed suddenly from 3,200 to 1,000 and the calculations were continued as usual.

In fig. 1 are shown the time traces of $Y-$component of velocity at five $X-$locations on the centre line $Y = 0.5, Z = 0.5$ for the case $IC2$. As expected, the flow adjusts quickly to the new value of $Re = 1{,}000$ near the top plate ($X = 0.00625$) and somewhat slowly at the bottom point ($X = 0.96875$). Finally, an asymptotic steady state results starting from the unsteady initial conditions $IC2$. A detailed comparison of the velocity fields was then made. All the three velocity components on the three centre lines of the cube were written with five significant figure accuracy and compared manually. They agreed for all the five significant figures, except for a few points near which the velocity changed sign and hence assumed extremely small values. Hence we can safely argue that two identical flow fields will result even though we start from two different initial fields, provided we wait sufficiently long.

After this preliminary experiment we move on to the more difficult test at Re =10,000 where the flow remains turbulent.

3.2 A Case of Turbulent Flow

Here again we obtain turbulent fields for $Re = 10,000$ but corresponding to two different initial conditions, viz., $IC1$, the zero initial condition and $IC2$ the one corresponding to Re = 3,200 and τ = 197.0. In fig. 2 are shown the time traces corresponding to $IC1$ at seven $X-$locations on the centre line $Y = 0.5, Z = 0.5$. The figure is divided into two parts (a) and (b) to avoid crowding. The corresponding traces for $IC2$ are shown in fig. 3(a) and 3(b). Here the initial time traces for Re = 3,200 are also shown for some of the locations.

Before we compare the two fields their convergence trends are studied. This is done by comparing the mean and the Reynolds stress components obtained for different signal lengths. The results are shown in figs. 4, 5 and 6. For the case of initial condition $IC1$, we take three signal segments of length T = 55, 110 and 165 units but all ending at τ = 211.4 units. Similarly for the case of initial condition $IC2$ the signal processing involves three segments of the same lengths of 55, 110 and 165 units but ending at τ = 442.92 units. We see from these three figures that the convergence of the mean velocity is very good in both the cases of initial conditions. The Reynolds stress components do not show the same degree of convergence. Convergence is specially slow in the primary vortex core (around X =0.5) for V_{rms} and near the bottom shear layer for $\overline{U'V'}$. But the same trend is seen in both the sets of data and even for the shortest signal length of 55 units the profile has the same qualitative features. Now we know if the signals were taken from the same source what kind of agreement we should roughly expect. This should help us in idenifying whether the two signals we are processing can be clubbed together as belonging to the same class, or are altogether different.

The comparison of the two results obtained starting from different initial conditions $IC1$ and $IC2$ is done next in figs. 7, 8 and 9. It is done along the two centre lines $(Y = 0.5, Z = 0.5)$ and $(X = 0.5, Z = 0.5)$ in the mid $Z-$plane. We have also included in these figures the experimental results of Prasad and Koseff [5]. Both the computational results were obtained by averaging for a period of 165 units. The experimental results were obtained by averaging for a different duration (see ref.[3] for detailed comparison) and have a different initial condition. But more importantly these results were obtained by an altogether different procedure and not by solving the Navier-Stokes equations. They are included here for additional comparison and also to highlight the difficulty one faces while comparing the numerical results with the experimental data, like in the present case.

From fig. 7 we see that the average velocity profiles - $\overline{V}$ along X and $\overline{U}$ along Y - are seen to agree well with each other and also generally with the experimental data. However, the peak value of $\overline{U}$ obtained experimentally near the downstream side wall is somewhat higher. But it should not bother us too much for the present comparison. The comparison of V_{rms} along X and U_{rms} along Y in fig. 8 is not as close. It is important to see that both the profiles have exactly the same qualitative features. When we move to fig. 9 to compare the Reynolds stress component $\overline{U'V'}$, the differences appear somewhat larger at the bottom wall and the right side wall (downstream) shear layers. But these are the locations where we had seen convergence to be the slowest. It was observed in ref. [3] that shear stress value being obtained by summing both positive and negative quantities (unlike the rms values) converges very slowly. The qualitative features of these two profiles are very close to each other and to the experimental data. However, quantitative differences where the convergence is not very good indicate that much longer averaging times are required.

To compare the frequency contents in the velocity signals obtained for the two initial conditions their spectra are compared in figs. 10 and 11. These are obtained by finding the discrete Fourier transforms of the respective signals. Further, the wiggles in the spectra were minimised to a certain extent by averaging the three spectra from 55 unit length signals. Thus a total signal length of 165 units was utilised. This averaging makes comparison of two spectra simpler. In fig. 10 are compared the spectra of U' components at two points (0.5, 0.36875, 0.5) and (0.5, 0.96875, 0.5) and in fig. 11 the V' spectra at the points (0.83125, 0.5, 0.5) and (0.96875, 0.5, 0.5). We see that all the four sets of spectra agree well in the frequency interval of 0.1 to 1. The low frequency limit here comes from the finte signal length of 55 units and hence f_{min}=1/55=0.0182 and the noise at the high frequency comes from the finite sampling rate $=1/(\Delta\tau)$ leading to a Nyquist frequency limit of $1/2(\Delta\tau) = 1/(2*0.05) = 10$. Subject to these limitations we see a good agreement of the corresponding spectra. It may also be pointed out that energy content in frequencies around one and above is extremely small. Thus it may be concluded that the frequency contents of these two sets of signals are the same.

4 Summary and Conclusions

Sensitivity of the numerically simulated flow field to initial conditions has been checked in this study. Even though the turbulent flows are known to be quite robust and independent of the initial conditions, this cannot be taken for granted for the numerically simulated turbulent flows. Flow in a three-dimensional lid driven cavity has been simulated using two distinct initial conditions - $IC1$ corresponds to starting from rest and $IC2$ to an unsteady flow field at Re=3,200.

As a preliminary exercise the flow at Re=1,000 was studied using these two initial conditions. Exactly the same asymptotic steady state was achieved starting from these two initial conditions. For the turbulent flow case at the Re=10,000 comparison of the two unsteady flow fields originating from two different initial conditions was done after estimating the degree of convergence achieved in these individual fields. Profiles of mean velocity, rms velocity and the Reynolds shear stress components were found to agree with each other qualitatively in the two fields. Even though the agreement was found to be very close for the mean velocity profiles, it was not close for the rms velocity components and still less for the shear stress profiles. Longer averaging times are required for the Reynolds stresses to settle down.

We have used only two initial conditions and stop here. If the two fields had not agreed, we could have had a more difficult problem in hand. This numerical exercise enhances the confidence in the numerical methods used to simulate the turbulent flow. This is required because of the uncertainties involved in any numerical procedure and also because the smallest scales in the flow, the Kolmogorov scales, have not been resolved [2]. This exercise also tells us what kind of precautions one has to take when comparing such numerical results with the experimantal data.

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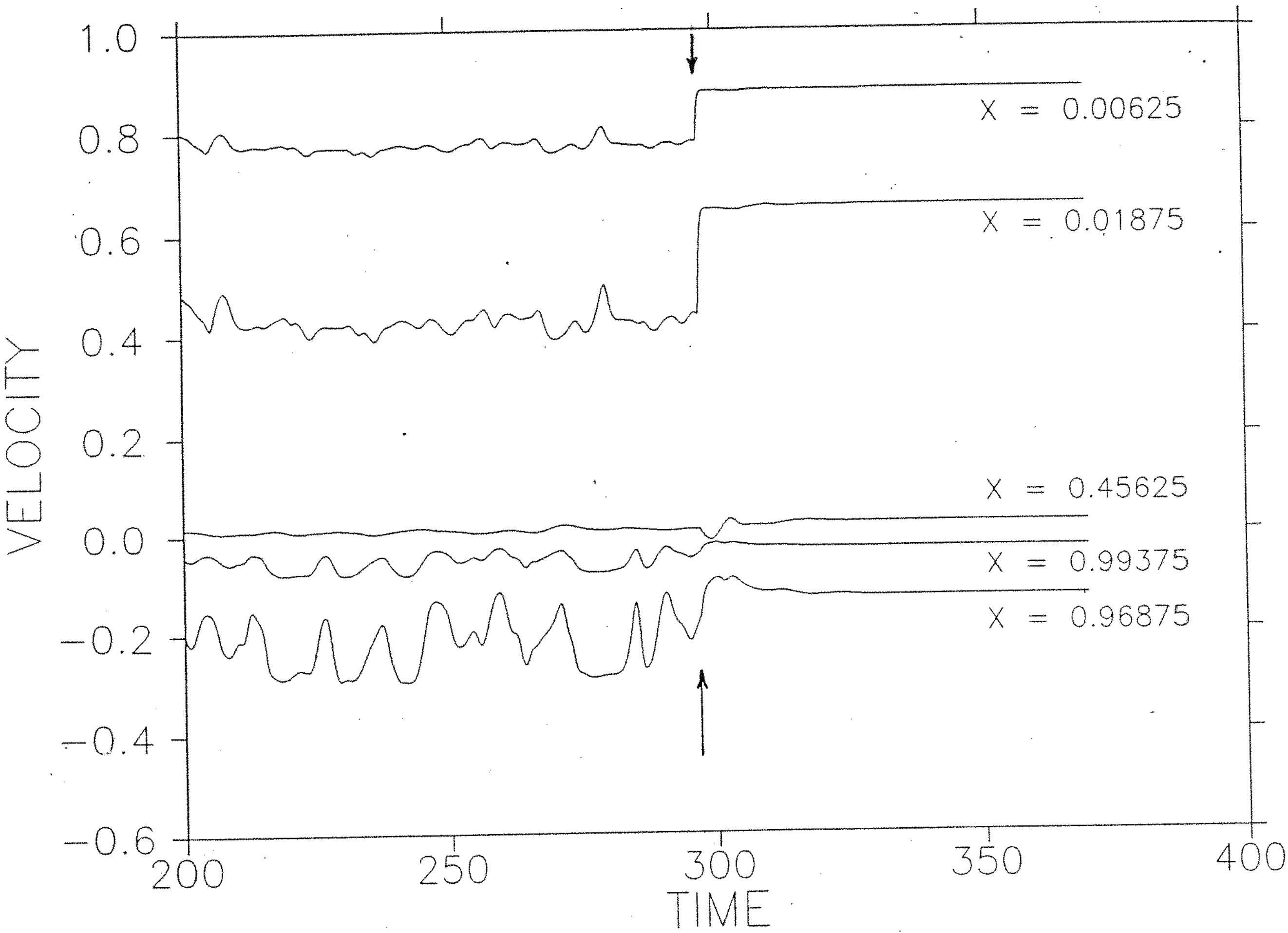


Figure 1: Time traces of $Y-$component of velocity for different $X-$locations on the line $Y = 0.5, Z = 0.5$; $Re = 1,000$. Initial condition $IC2$ from $Re = 3,200$ applied at time $\tau = 297.1$ units.

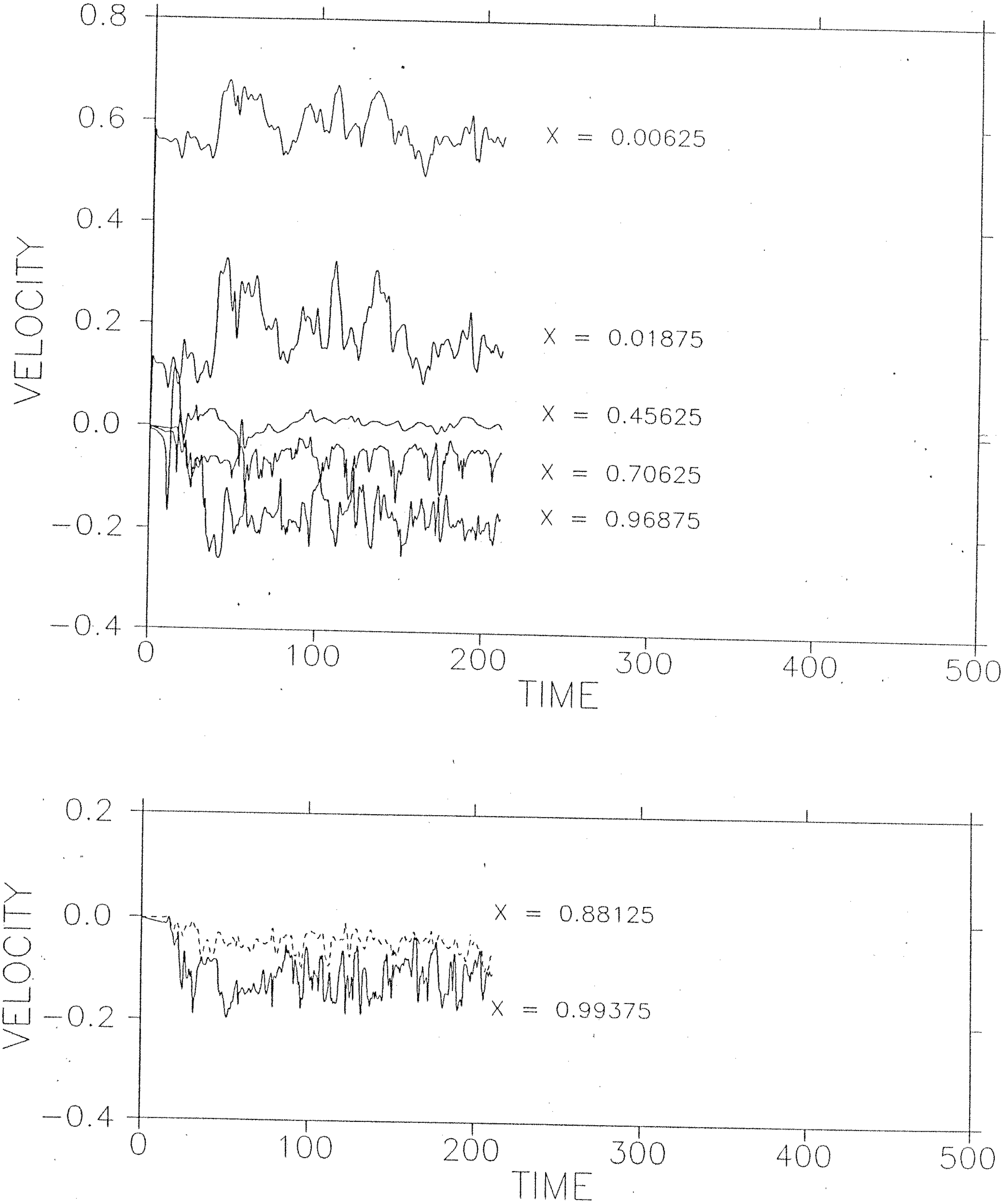


Figure 2: Time traces of $Y-$component of velocity for different $X-$locations on the line $Y = 0.5, Z = 0.5$; $Re = 10,000$. Initial condion $IC1$ corresponds to starting from rest. Figure split into (a) and (b) to avoid crowding.

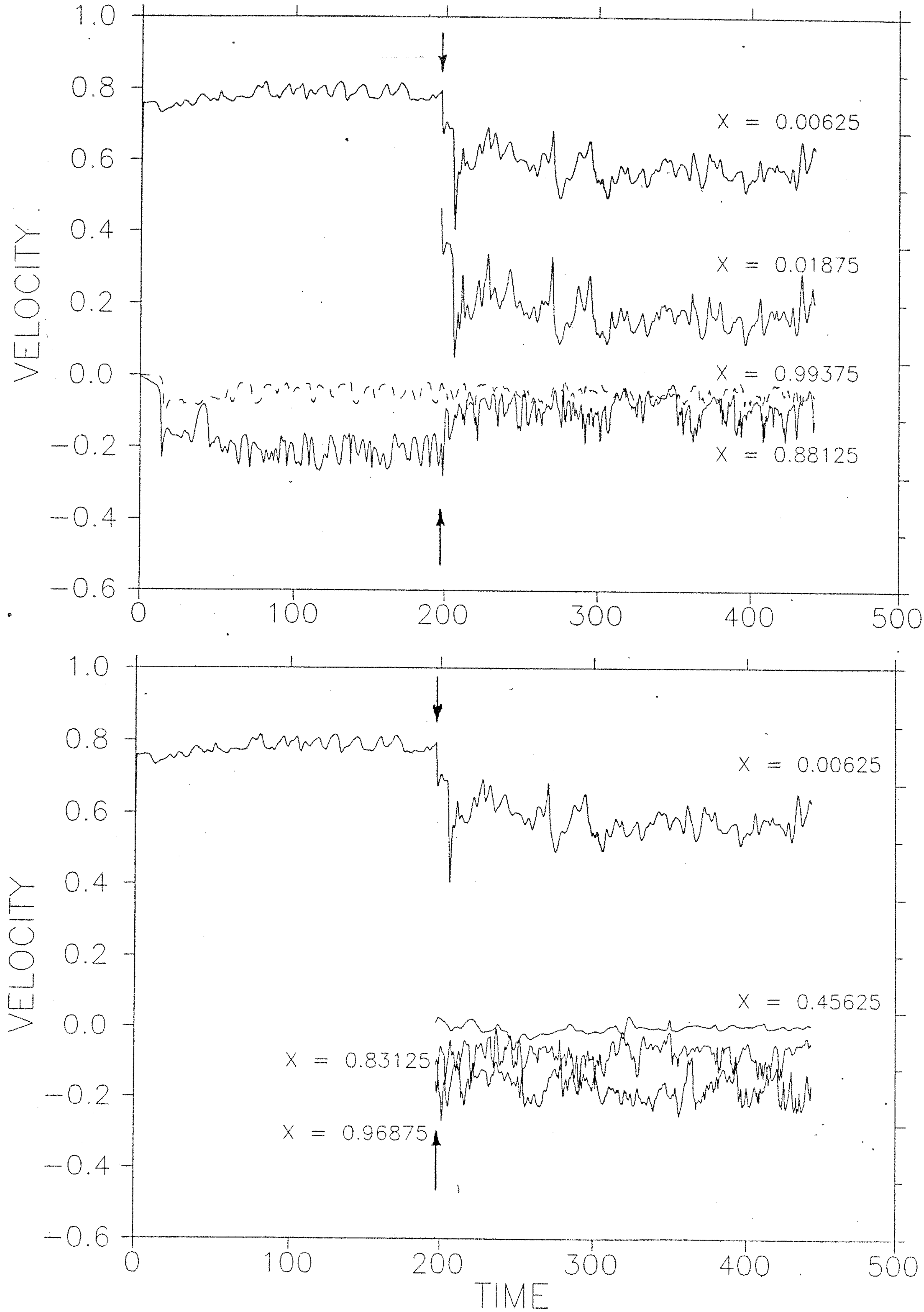


Figure 3: Time traces of $Y-$component of velocity for different $X-$locations on the line $Y = 0.5, Z = 0.5$; $Re = 10,000$. Initial condition $IC2$ from $Re = 3,200$ applied at time $\tau = 197.0$ units. Figure split into (a) and (b) to avoid crowding.

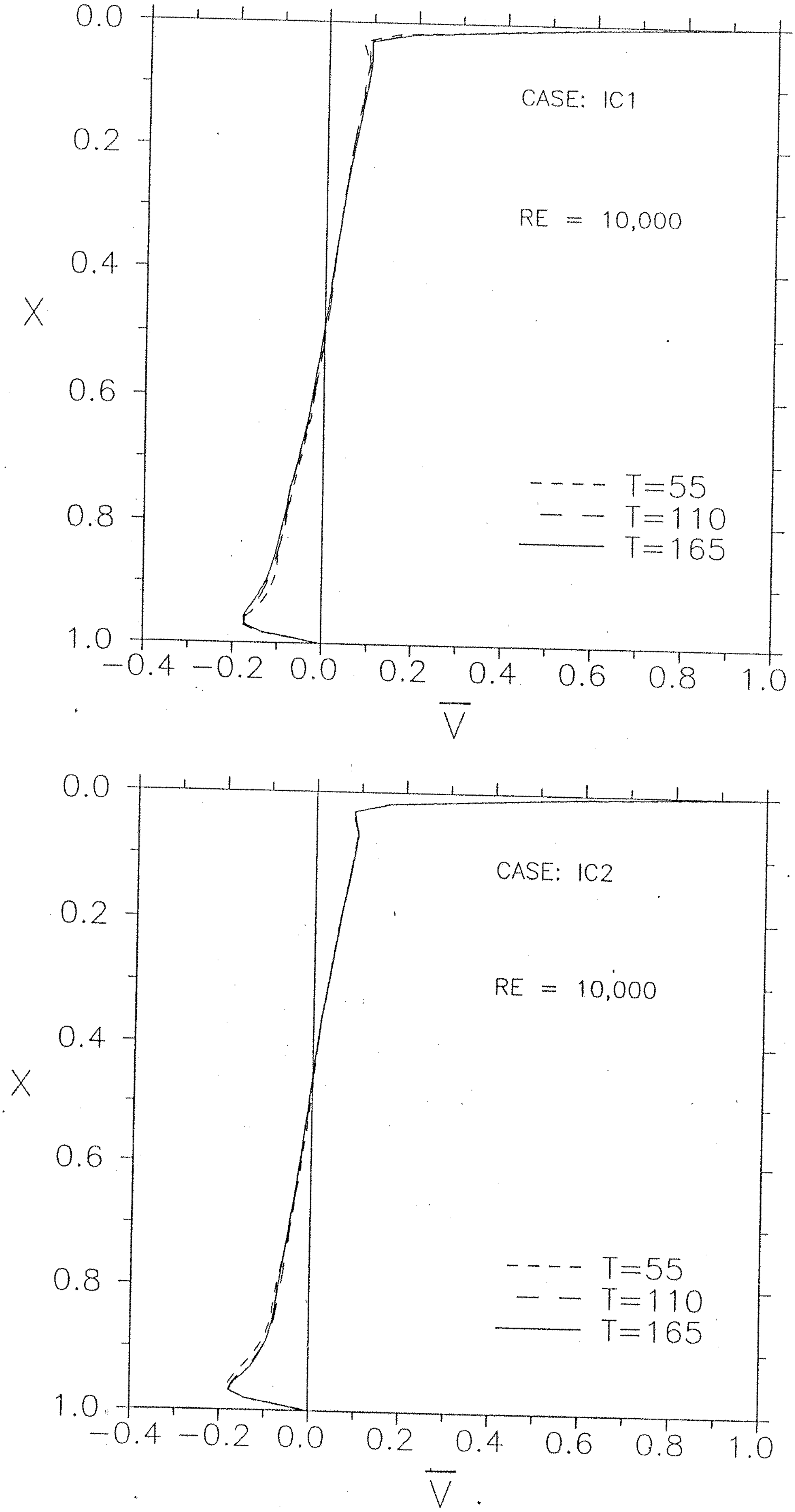


Figure 4: Mean velocity profiles of $Y-$component of velocity along the line $Y = 0.5$, $Z = 0.5$ compared for three lengths of data, T; $Re = 10,000$. (a) case $IC1$ starting from rest, (b) case $IC2$.

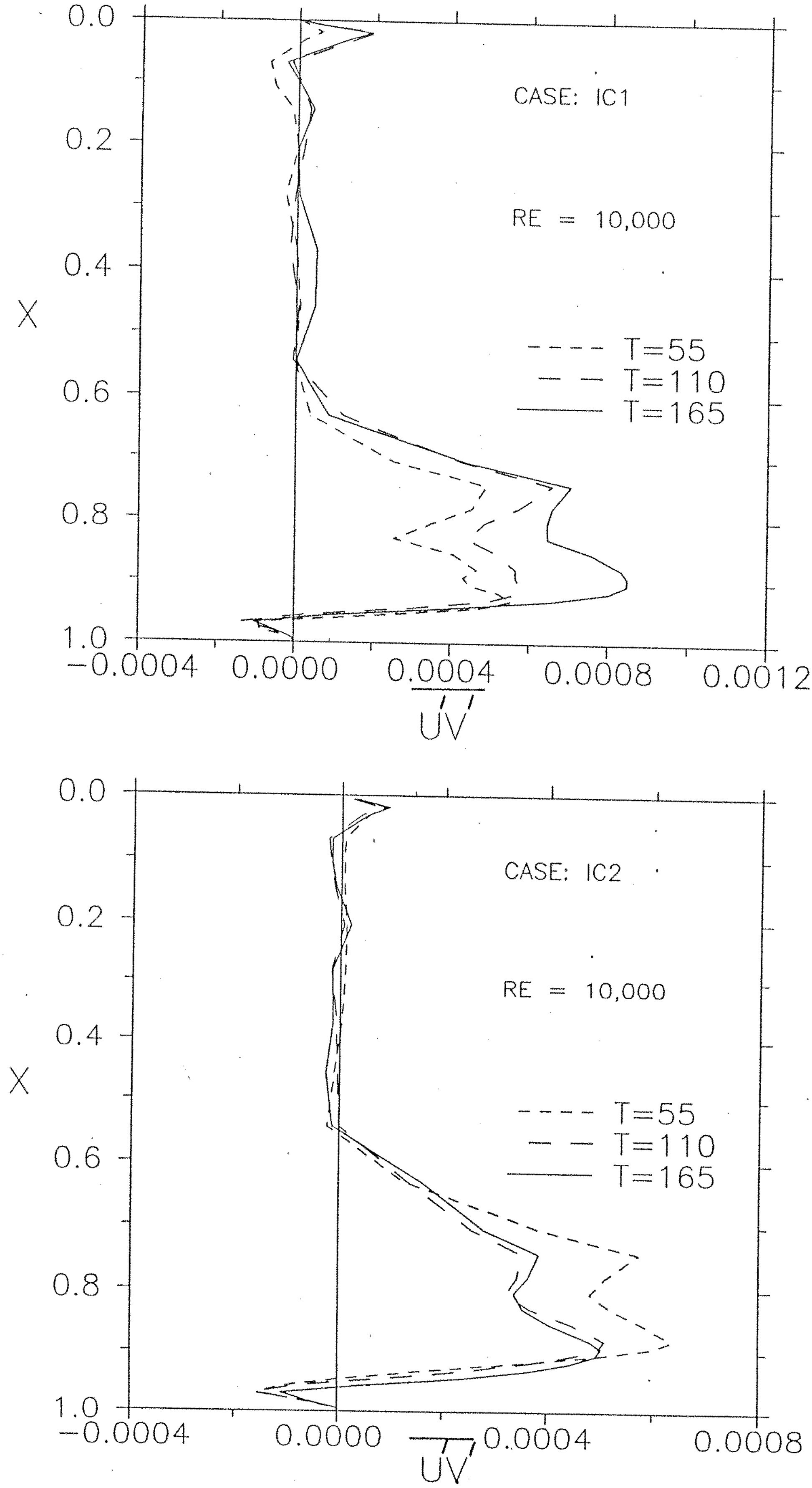


Figure 6: Profiles of $\overline{U'V'}$ along the line $Y = 0.5, Z = 0.5$ compared for three lengths of data, T; $Re = 10,000$. (a) case $IC1$ starting from rest, (b) case $IC2$.

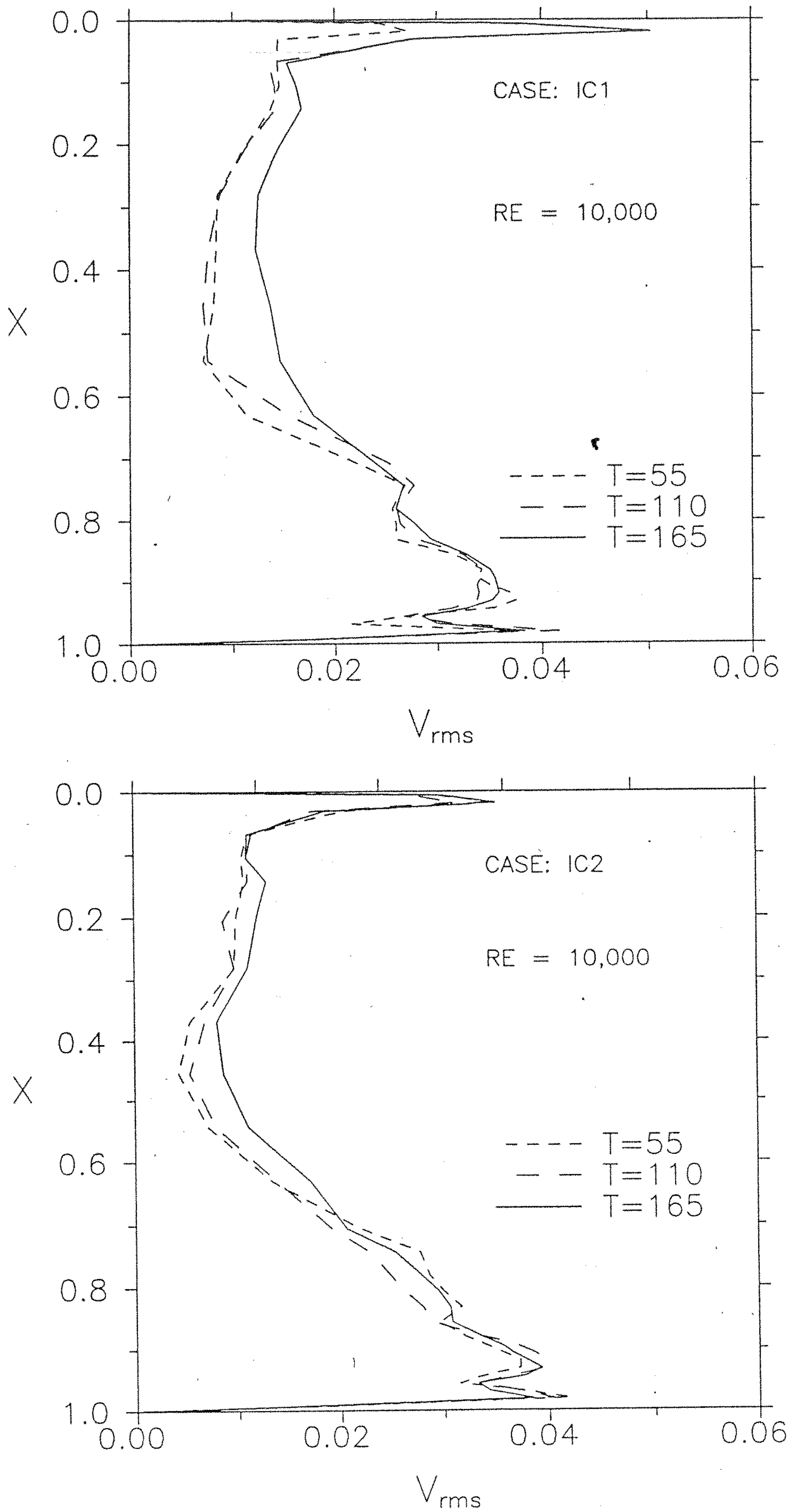


Figure 5: RMS velocity profiles of Y−component of velocity along the line $Y = 0.5, Z = 0.5$ compared for three lengths of data, T; $Re = 10,000$. (a) case $IC1$ starting from rest, (b) case $IC2$.

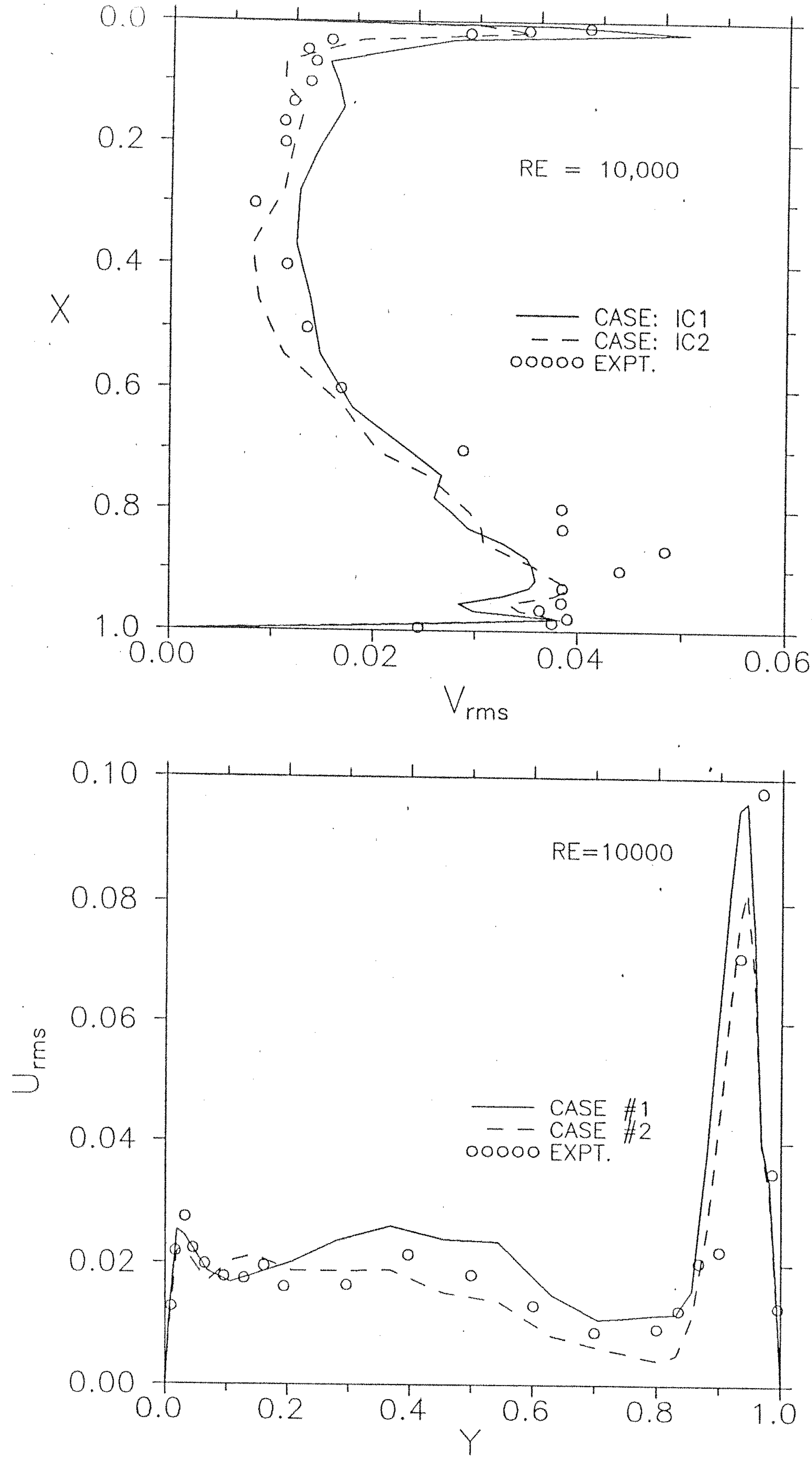


Figure 8: RMS velocity profiles for $IC1$ and $IC2$ compared with each other and with the experimental data, $Re = 10,000$. The length of data $T = 165$ units for both $IC1$ and $IC2$. (a) V_{rms} vs. X along $Y = 0.5, Z = 0.5$, (b) U_{rms} vs. Y along $X = 0.5, Z = 0.5$.

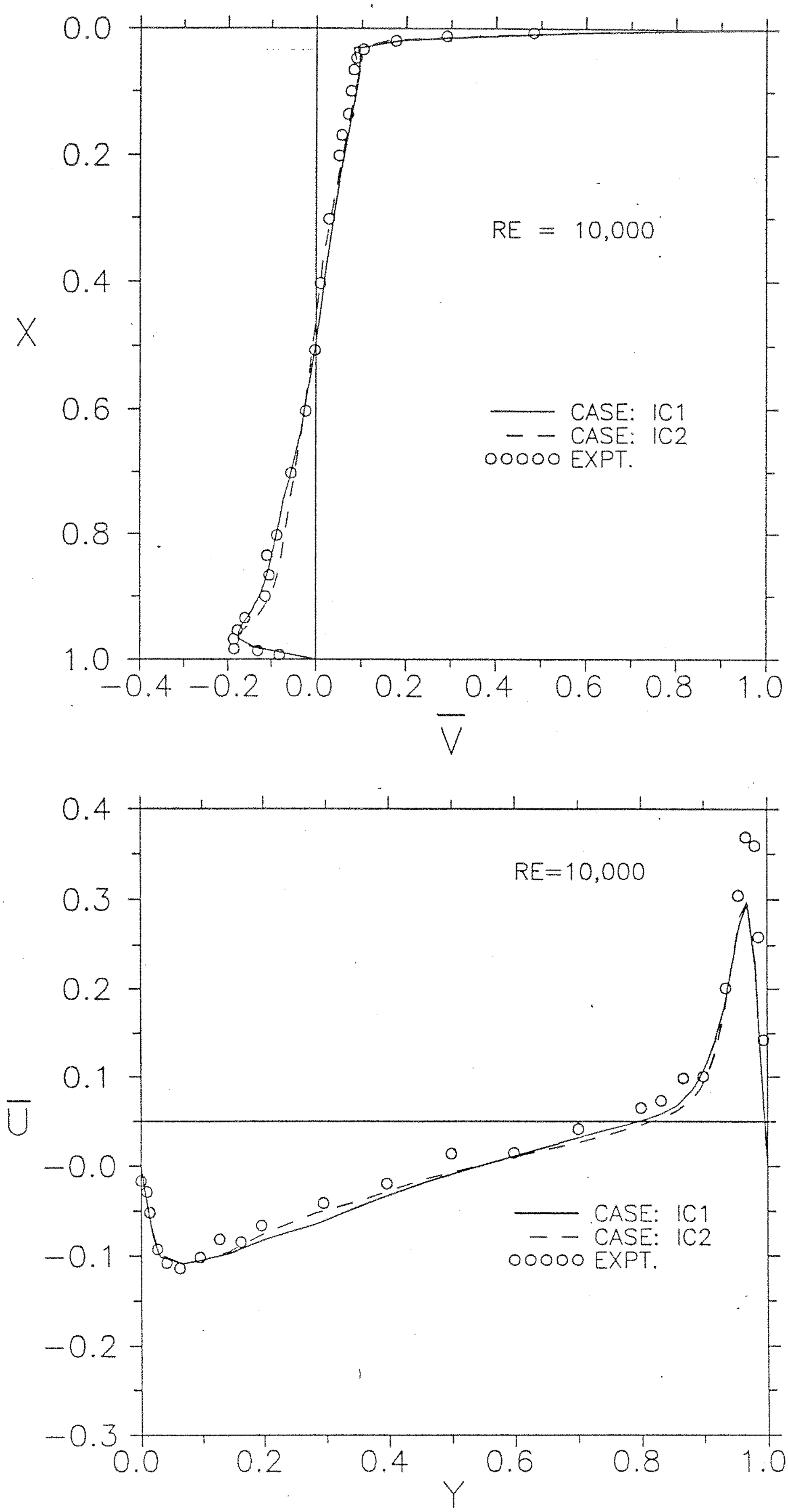


Figure 7: Mean velocity profiles for $IC1$ and $IC2$ compared with each other and with the experimental data, $Re = 10,000$. The length of data $T = 165$ units for both $IC1$ and $IC2$. (a) $\overline{V}$ vs. X along $Y = 0.5, Z = 0.5$, (b) $\overline{U}$ vs. Y along $X = 0.5, Z = 0.5$.

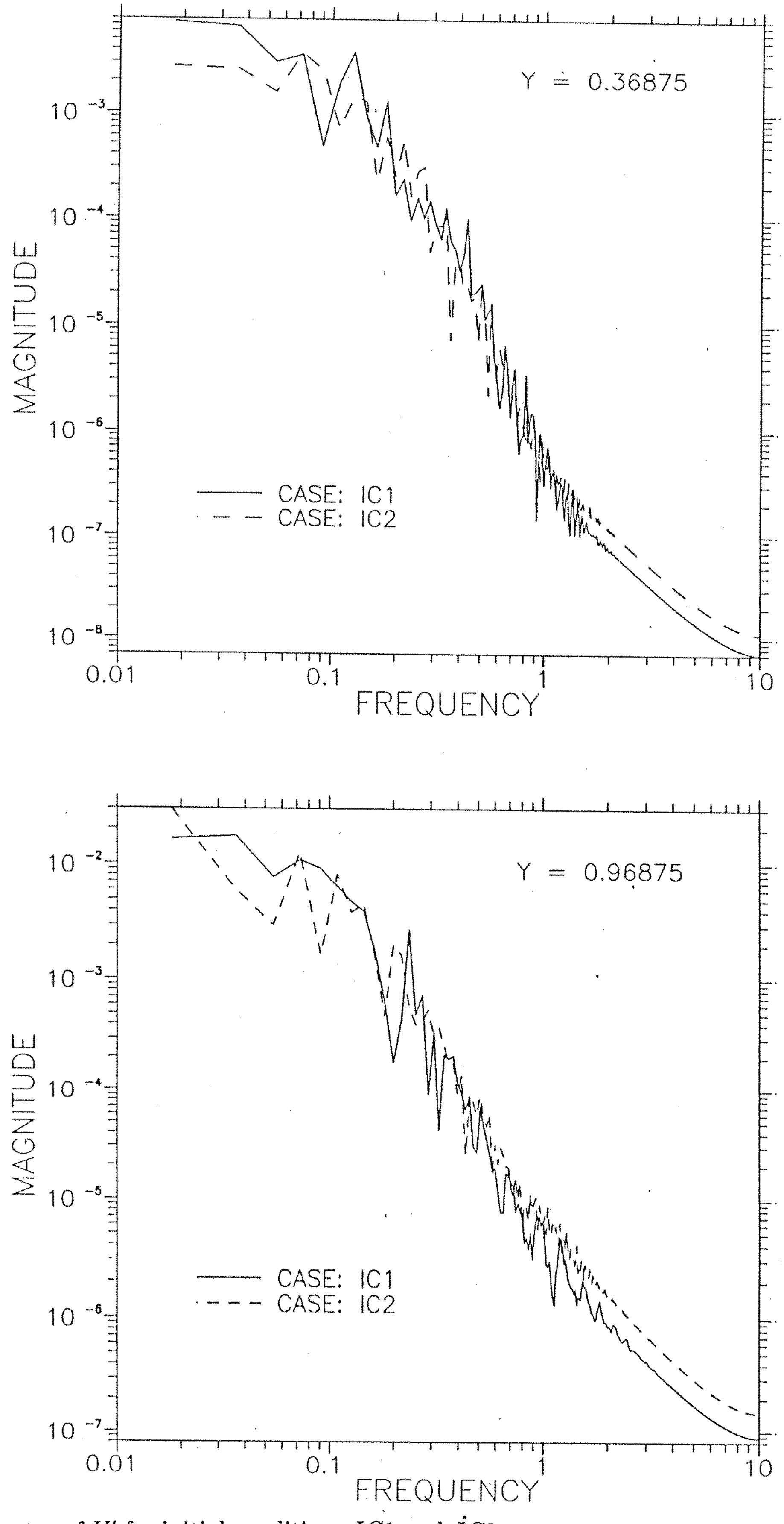


Figure 10: Spectra of U' for initial conditions $IC1$ and $IC2$ compared with each other at two locations on the line $X = 0.5, Z = 0.5$. The spectra are averaged over three segments of 55 unit length each. $Re = 10,000$. (a) $Y = 0.36875$, (b) $Y = 0.96875$.

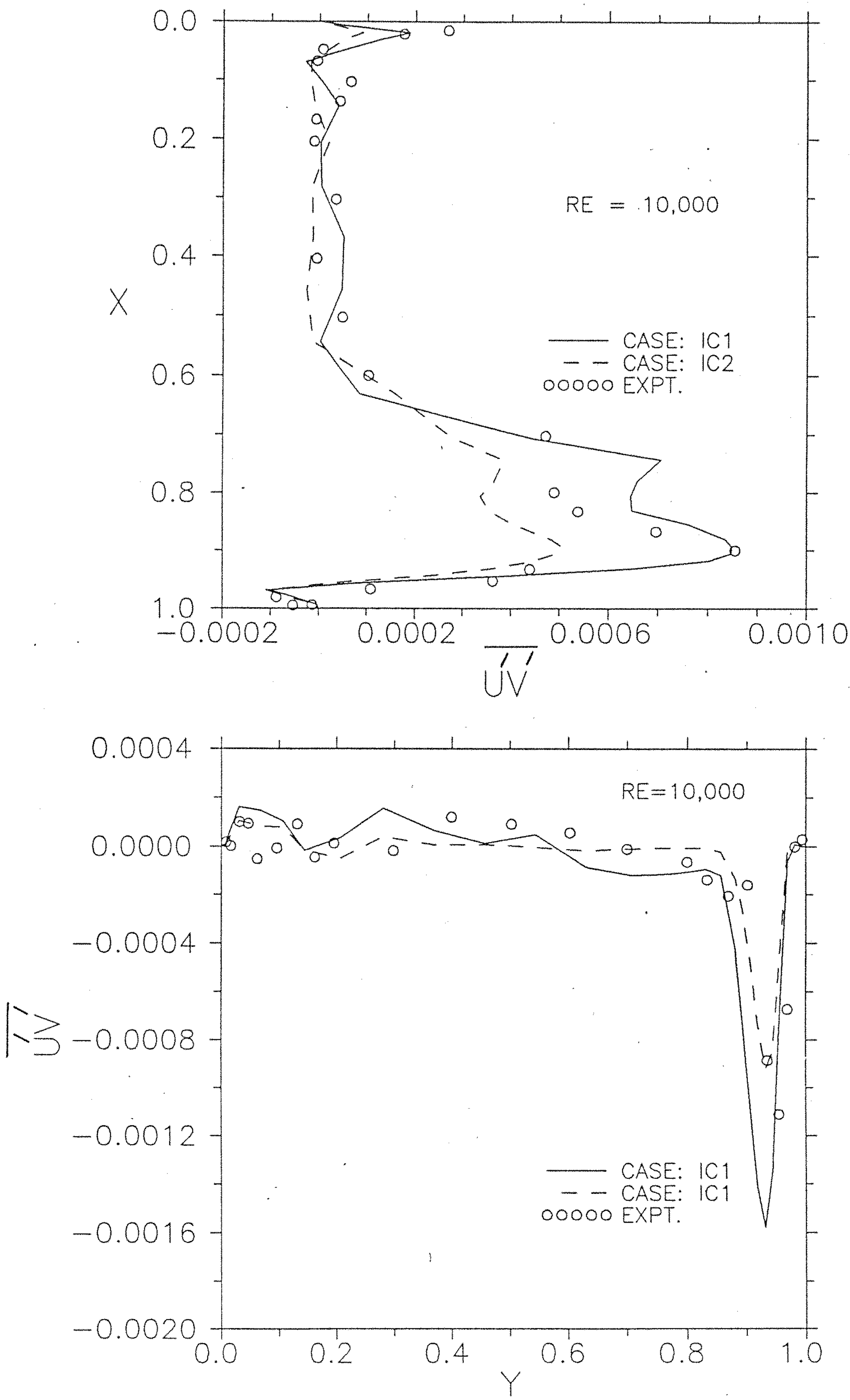
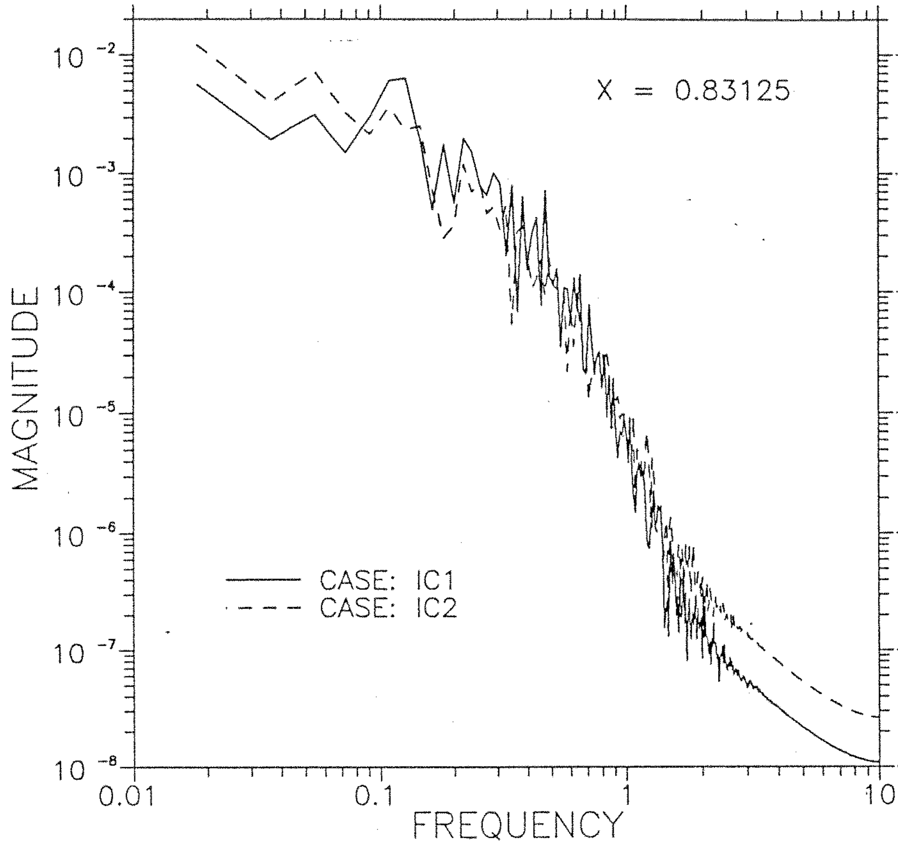


Figure 9: Profiles of $\overline{U'V'}$ for $IC1$ and $IC2$ compared with each other and with the experimental data, $Re = 10,000$. The length of data $T = 165$ units for both $IC1$ and $IC2$. (a) $\overline{U'V'}$ vs. X along $Y = 0.5, Z = 0.5$, (b) $\overline{U'V'}$ vs. Y along $X = 0.5, Z = 0.5$.



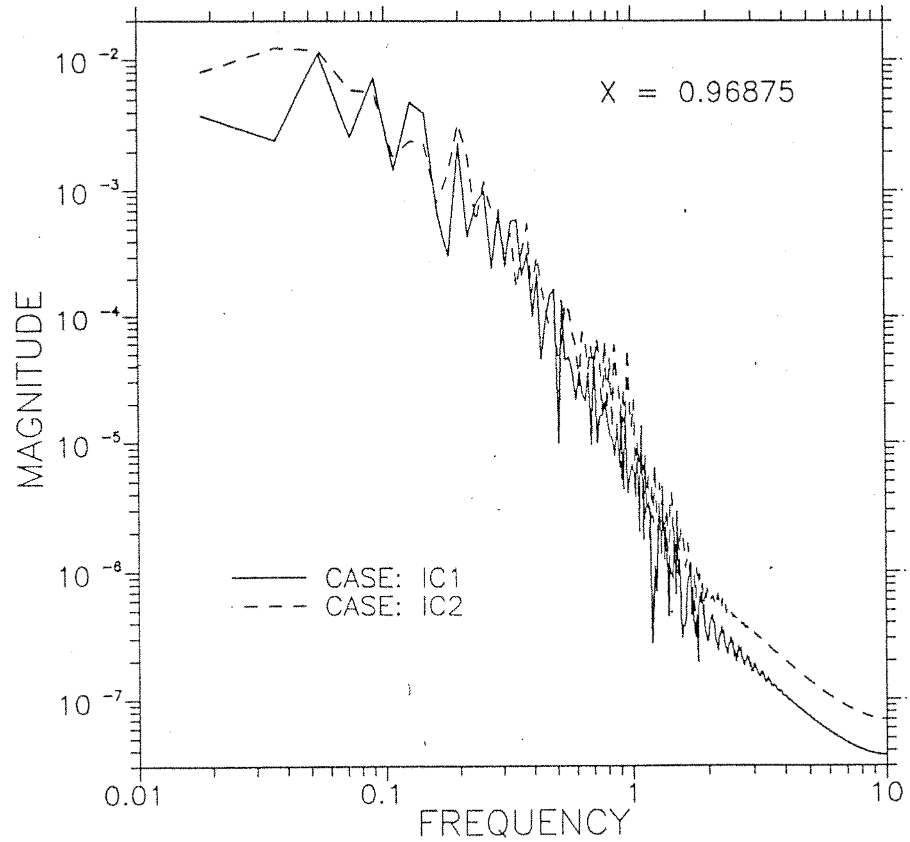


Figure 11: Spectra of V' for initial conditions $IC1$ and $IC2$ compared with each other at two locations on the line $Y = 0.5, Z = 0.5$. The spectra are averaged over three segments of 55 unit length each. $Re = 10,000$. (a) $X = 0.83125$, (b) $X = 0.96875$.

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Abstract

Sensitivity of the numerically simulated flow field to the initial conditions has been checked by comparing the flows obtained by starting from different initial conditions. Flow in a three-dimensional cubical cavity driven by the motion of the lid is studied at two different Reynolds numbers Re = 1,000 and Re = 10,000. The two initial conditions used are $IC1$: starting from rest and $IC2$: corresponding to the unsteady flow at Re = 3,200. Identical steady flow at Re = 1,000 was achieved with these initial conditions, $IC1$ and $IC2$. The flow at Re = 10,000 is unsteady (turbulent) and hence comparison of the two fields obtained starting from $IC1$ and $IC2$ is not straight forward. Hence the convergence of the two fields was studied to see what kind of uncertainties exist. Then the mean velocity profiles, *rms* velocity profiles, and the shear stress profiles on selected lines and also turbulence spectra at a few points were compared. It was found that the two turbulence fields obtained starting from $IC1$ and $IC2$ were generally in qualitative agreement with one another. However, there are quantitative differences in the Reynolds shear stresses, which we believe show that much longer averaging times are required for these quantities to settle down.